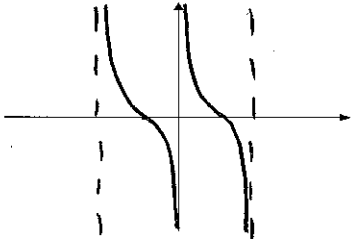
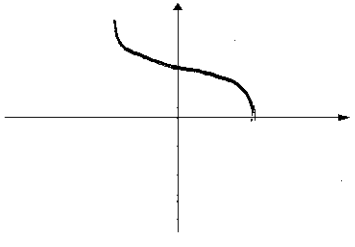
Sketch the graphs. **For periodic functions, sketch at least 2 periods.** **SCORE: ____ / 12 PTS**

You only need to get the general position and shape correct. Do NOT plot points.

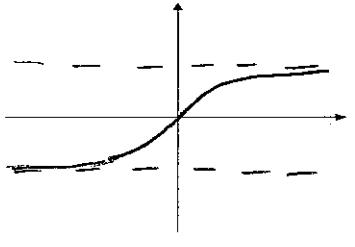
[a] $y = \cot x$



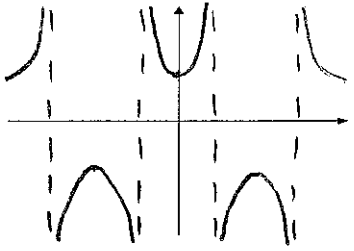
[b] $y = \cos^{-1} x$



[c] $y = \arctan x$



[d] $y = \sec x$



Fill in the blanks. Write "DNE" if the question has no answer. **SCORE: ____ / 28 PTS**

$\arcsin\frac{\sqrt{2}}{2} =$ $\frac{\pi}{4}$

As $x \to \frac{\pi}{2}^+$, $\sec x \to$ $-\infty$

$\arctan(-\sqrt{3}) =$ $-\frac{\pi}{3}$

$\cos^{-1}(-\frac{\sqrt{3}}{2}) =$ $\frac{5\pi}{6}$

$\arccos(\cos\frac{2\pi}{3}) =$ $\frac{2\pi}{3}$

$\sin(\sin^{-1}\frac{3}{2}) =$ DNE

$\arctan(\tan\frac{3\pi}{4}) =$ $-\frac{\pi}{4}$

$\tan(\tan^{-1}7) =$ 7

The range of $f(x) = \arccos x$ is $[0, \pi]$

The domain of $f(x) = \sin^{-1} x$ is $[-1, 1]$

The domain of $f(x) = \tan x$ is $x \neq \frac{\pi}{2} + n\pi,\ n \in \mathbb{Z}$

The range of $f(x) = \sec x$ is $(-\infty, -1] \cup [1, \infty)$

The **equations** of the asymptotes of $f(x) = \arctan x$ are

$y = \pm\frac{\pi}{2}$

The **equations** of the asymptotes of $f(x) = \csc x$ are

$x = n\pi,\ n \in \mathbb{Z}$

Graph 2 periods of the function $y = -4\cos(\frac{4}{3}x + \frac{11\pi}{6}) - 3$.

SCORE: ____ / 16 PTS

Find the coordinates of the 9 points discussed in lecture, corresponding to 2 complete periods, starting at the phase shift.
Label all $x-$ and $y-$ values for the 9 points on the appropriate axes, using a consistent scale for each axis.

MID $= -3$

AMP $= |-4| = 4$

MAX $= -3+4 = 1$

MIN $= -3-4 = -7$

PERIOD $= \frac{2\pi}{\frac{4}{3}} = 2\pi \cdot \frac{3}{4} = \frac{3\pi}{2}$

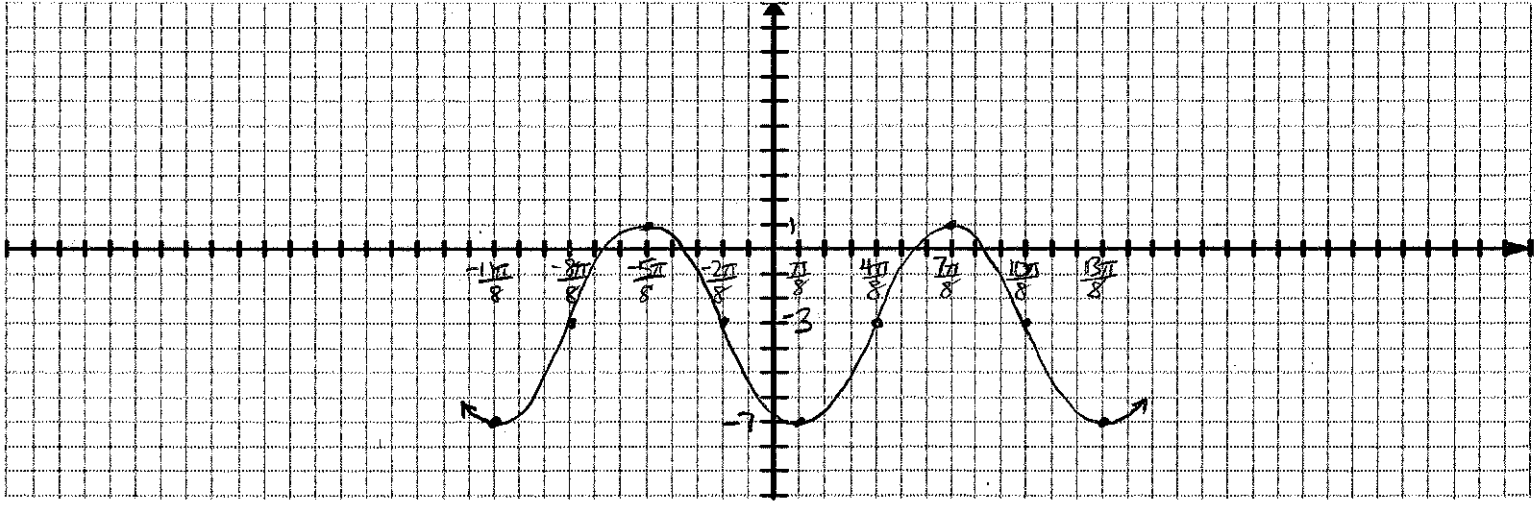
$\frac{1}{4}$ PERIOD $= \frac{3\pi}{8}$

START $\frac{4}{3}x + \frac{11\pi}{6} = 0 \rightarrow \frac{4}{3}x = -\frac{11\pi}{6} \rightarrow x = -\frac{11\pi}{6_2} \cdot \frac{3}{4} = -\frac{11\pi}{8}$

POINTS:

$(\frac{-11\pi}{8}, -7)$ $(\frac{-8\pi}{8}, -3)$ $(\frac{-5\pi}{8}, 1)$ $(\frac{-2\pi}{8}, -3)$ $(\frac{\pi}{8}, 7)$

$(\frac{4\pi}{8}, -3)$ $(\frac{7\pi}{8}, 1)$ $(\frac{10\pi}{8}, -3)$ $(\frac{13\pi}{8}, 7)$

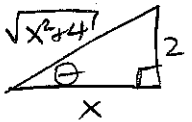


Simplify the following expressions completely. Show proper reasoning to justify your answer. **SCORE: ____ / 14 PTS**

[a] $\csc(\arctan\frac{2}{x})$, where $x > 0$

$\theta = \arctan\frac{2}{x}$

$\tan\theta = \frac{2}{x}$



$\csc(\arctan\frac{2}{x}) = \csc\theta = \frac{\sqrt{x^2+4}}{2}$

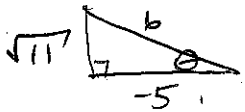
[b] $\tan(\cos^{-1}(-\frac{5}{6})) = \tan\theta = -\frac{\sqrt{11}}{5}$

$\theta = \cos^{-1}(-\frac{5}{6})$

$\cos\theta = -\frac{5}{6}$, $\theta \in [0, \pi]$

I.E. θ IN Q_1 OR Q_2

$\cos\theta < 0 \rightarrow \theta$ IN Q_2



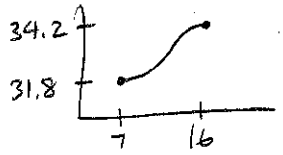
Due to a malfunction, the temperature in a lab freezer has been behaving like a sinusoidal function. **SCORE: _____ / 14 PTS**

At 12:07pm, the temperature reached a low of $31.8^\circ F$, then increased until it reached a high of $34.2^\circ F$ at 12:16pm.

[a] Find an equation for the temperature in the freezer at t minutes after noon.

$\text{MID} = \frac{34.2+31.8}{2} = 33$

$\text{AMP} = \frac{34.2-31.8}{2} = 1.2$



START $t = 7$

PERIOD $2(16-7) = 18 = \frac{2\pi}{B} \longrightarrow B = \frac{2\pi}{18} = \frac{\pi}{9}$

$-1.2\cos\frac{\pi}{9}(t-7)+33$

[b] Find the temperature in the freezer at 1pm. (Round your answer to 1 decimal point.)

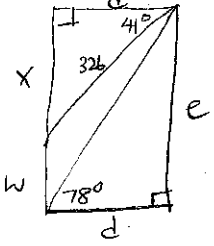
$-1.2\cos\frac{\pi}{9}(60-7)+33 = 31.9^\circ$

A 326 foot cable connects the roofs of two buildings. **SCORE: ____ / 16 PTS**

From the base of the west building, the angle of elevation to the roof of the east building is 78°.

From the roof of the east building, the angle of depression to the roof of the west building is 41°.

[a] Find the height of each building. **You may need to calculate other information that is not explicitly requested.** Round your answers to the nearest integer. **You may only use techniques covered in lecture so far.**



$\cos 41^\circ = \frac{d}{326} \rightarrow d = 326 \cos 41^\circ = 246$

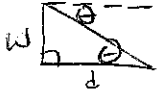
$\sin 41^\circ = \frac{x}{326} \rightarrow x = 326 \sin 41^\circ = 214$

$\tan 78^\circ = \frac{e}{d} \rightarrow e = d \tan 78^\circ = 246 \tan 78^\circ = 1157$

$w = e - x = 1157 - 214 = 943$

EAST 1157 FT, WEST 943 FT

[b] Find the angle of depression from the roof of the west building to the base of the east building. Round your answer to the nearest integer.



$\tan \theta = \frac{w}{d} \rightarrow \theta = \tan^{-1} \frac{w}{d} = \tan^{-1} \frac{943}{246} = 75^\circ$